

Comprehensive Evaluation of Students' Mathematics Performance Based on Fuzzy Clustering Analysis

Ganzhou Wu

School of science, Guangdong University of Petrochemical Technology, Maoming 525000, China

Abstract: It is introduced the fuzzy clustering analysis method to construct a multidimensional comprehensive evaluation model for students mathematics grades. Firstly, summarize the basic principles of fuzzy clustering analysis and related theories of educational evaluation; Secondly, 86 students from two classes were selected as the research subjects, and data on seven indicators including unit tests, mid-term and final grades, homework completion rates, and classroom participation were collected; Once again, the fuzzy C-means (FCM) algorithm was used to perform cluster analysis on students, and the optimal number of clusters was determined to be 3, resulting in three typical student profiles: 30 stable and excellent students, 35 potential and developmental students, and 21 students with weak foundations; Finally, based on the clustering results, differentiated teaching strategies and dynamic tracking mechanisms are proposed. Comparative analysis shows that the fuzzy clustering method can more scientifically reveal the structural differences in student performance compared to the traditional average segmentation.

Keywords: Fuzzy clustering analysis; Performance evaluation; Fuzzy C-means.

1. Introduction

Cluster analysis, as an unsupervised learning method, is widely used in fields such as student academic performance classification and learning behavior pattern recognition. Qadirs review studies indicate that K-means is the most commonly used clustering algorithm between 2013 and 2023. Hasan [1] et al. used K-means clustering based on 1200 student records to classify students into three categories: active learners, passive learners, and high-risk students, effectively achieving academic warning. In the field of basic education, research has used K-means to cluster multidimensional performance data of junior high school students and divide them into different levels of performance [2]. The Davies Bouldin index has verified good discrimination, providing a basis for hierarchical teaching. These studies indicate that cluster analysis has evolved from simple score classification to a multidimensional analysis tool that covers academic performance and learning behavior.

Fuzzy clustering allows samples to belong to multiple categories with different degrees of membership, making it more suitable for common boundary blurring situations in educational evaluation. The fuzzy C-means (FCM) algorithm proposed by Bezdek laid the foundation for subsequent research. In the field of education, Bustomi et al. conducted FCM clustering using academic average score, attitude, motivation, and attendance rate as indicators in the junior high school context. The results showed that the fuzzy partition coefficient and Silhouette score were both better, and the membership information of each student in different clusters provided richer details for personalized diagnosis [3]. Rustam and Usman used FCM to cluster students course learning outcomes and found that fuzzy clustering can reveal "potential transitions" and "overlapping boundaries" that traditional rigid partitioning cannot capture, providing methodological support for interpretable learning trajectory modeling [4]. The number of cases in which domestic scholars apply cluster analysis to academic performance analysis is gradually increasing. Shi Zehong conducted a study on grade prediction based on multi-source data and used

K-means clustering to reduce the dimensionality of features [5]. Xie Yuting explored the correlation between courses based on student performance and the evaluation of students comprehensive quality, using systematic clustering method to analyze the similarity between courses [6]. Wang Shaoxuan established a framework for analyzing middle school students grades based on fuzzy reasoning models, introducing fuzzy logic into grade grading [7]. Zhao Jinlus masters thesis "Research on Comprehensive Evaluation and Application of Middle School Academic Achievement Based on Fuzzy Cluster Analysis" selected student achievement data from three grades of a certain middle school, used fuzzy clustering to divide students into three categories, and proposed teaching suggestions [8].

2. The mathematical principle of fuzzy C-means algorithm

The fuzzy C-means algorithm was proposed by Bezdek in 1981 and is currently the most widely used fuzzy clustering algorithm [9].

Set a sample set $X = \{x_1, x_2, \dots, x_n\}$, with each sample having a feature value $c (2 \leq c \leq n)$ to divide it into fuzzy clusters.

The objective function of FCM is:

$$J_m(U, V) = \sum_{i=1}^c \sum_{j=1}^n u_{ij}^m \|x_j - u_i\|^2 \quad (1)$$

Construct Lagrange function:

$$L = \sum_{i=1}^c \sum_{j=1}^n u_{ij}^m d_{ij}^2 + \sum_{j=1}^n \lambda_j \left(\sum_{i=1}^c u_{ij} - 1 \right) \quad (2)$$

$$\text{st.} \quad \sum_{i=1}^c u_{ij} = 1 \quad (3)$$

where

$$d_{ij} = \|x_j - v_i\| \quad (4)$$

Find the partial derivative u_{ij} and make it zero:

$$\frac{\partial L}{\partial u_{ij}} = mu_{ij}^{m-1} d_{ij}^2 + \lambda_j = 0 \quad (5)$$

We obtain:

$$u_{ij} = \left(-\frac{\lambda_j}{md_{ij}^2} \right)^{\frac{1}{m-1}} \quad (6)$$

Using constraint elimination $\sum_{i=1}^c u_{ij} = 1$.

$$u_{ij} = \frac{1}{\sum_{k=1}^c \left(\frac{d_{ij}}{d_{kj}} \right)^{\frac{2}{m-1}}} \quad (7)$$

Find the partial derivative v_i and make it zero:

$$\frac{\partial J_m}{\partial v_i} = \sum_{j=1}^n u_{ij}^n \cdot 2(v_i - x_j) = 0 \quad (8)$$

By solving the above equation, we obtain

$$v_i = \frac{\sum_{j=1}^n u_{ij}^m x_j}{\sum_{j=1}^n u_{ij}^m} \quad (9)$$

Experimental results

First of all, The category boundaries in educational

evaluation are often vague. For example, there is no absolute boundary between "excellent" and "good", and a student who scores 85 may achieve excellent levels in certain abilities while only being good in others. Traditional hard clustering, such as K-means, forces each sample to belong to one and only one category, losing membership information. Let fuzzy clustering give a membership degree of 0.6 for a student belonging to the "excellent class" and 0.4 for belonging to the "good class". This means that the student mainly exhibits excellent characteristics, but also has some good characteristics. Teachers can adopt a more hierarchical teaching strategy based on this: focusing on the requirements of excellent classes, while appropriately supplementing the consolidation exercises of good classes. However, hard clustering can only classify it as an excellent class, which may overlook its weak links.

Due to the fact that process indicators have not been included in the formal evaluation system, teachers' understanding of students' learning status mainly relies on exam scores. Exam scores have a lag: when an exam score drops significantly, problems may have accumulated for several weeks. If fuzzy clustering is combined with regularly updated process data (such as weekly homework completion rate and classroom exercise accuracy), it can identify students who have "continuously decreased process dimensions but not significantly decreased knowledge dimensions" in the early stage and provide early warning. The sample comes from 86 students from a certain school.

Table 1. Comprehensive evaluation indicators for mathematics grades

Dimension	Indicator	Symbol	Data source	Method	Unit
knowledge	Unit test average	x_1	4 unit tests	mean	100 points
knowledge	Mid term results	x_2	Mid term results	Original Score	100 points
knowledge	final grade	x_3	final grade	Original Score	100 points
ability	scoring rate	x_4	scoring rate	scoring rate	100 points
ability	Homework completion rate	x_5	12 Homework	completion rate	Percentage
process	Practice accuracy	x_6	8 small exercises	ractice accuracy	Percentage
process	class participation	x_7	Teacher Rating	Monthly average	1-5

Due to the different dimensions and ranges of values of the indicators in the above table, standardization is required. This article adopts Z-score standardization: after standardization, the mean of each indicator is 0, and the standard deviation is 1.

Take $c=2, 3, 4$, and 5 respectively, run the FCM algorithm ($m=2$, $\varepsilon=0.001$, randomly initialize 10 times and choose the best), calculate the partition coefficient PC and partition entropy PE. The results are shown in Table 2.

Table 2. the results of obtaining PC and PE with different c values

c	Partition coefficient (PC)	Partition Entropy (PE)
2	0.83	0.28
3	0.81	0.34
4	0.76	0.46
5	0.73	0.52

Although the PC of $c=2$ is slightly higher than that of $c=3$, the increase in PE is greater at $c=3$, and from an educational explanatory perspective, the three categories are more reasonable, so $c=3$ is chosen. From an educational perspective,

dividing students into three categories (excellent, potential, weak) is also the granularity that teachers can most easily understand and apply.

Set the fuzzy index $m=2$, the termination error $\varepsilon=0.001$, and the maximum number of iterations is 100. Randomly initialize the membership matrix, repeat 10 times, and select the result with the smallest objective function value.

The algorithm converges after the 12th iteration, and the objective function value decreases from the initial 312.5 to 51.3958. Figure 1 shows the decreasing curve of the objective function with the number of iterations

The final membership matrix U is 86×3 . Each row corresponds to a sample, each column corresponds to a category, and the sum of each row is 1. Table 3 shows the membership degrees of some students.

According to Table 3, the membership degree of most samples is significantly high in a certain category (≥ 0.7), but there are also some edge samples (such as student number 6, with membership degrees of 0.30, 0.66, and 0.04 for the three categories respectively, although ultimately classified as potential type, the membership degree of excellent type also

reaches 0.30; student number 10, weak type 0.61, excellent type 0.34, with obvious ambiguity). This is precisely where the advantage of fuzzy clustering over hard clustering lies - it preserves students degree of belonging to different categories, providing teachers with more refined diagnostic information on their learning situation. By calculating the cluster centers for each category, students can be classified into three types.

Excellent type (30 people): All indicators are significantly higher than the overall average, with a comprehensive application score rate of 84.3% and excellent process dimensions (homework completion rate of 94.7%, participation rate of 4.42).

Potential type (35 people): Moderate knowledge score (average score 71-76), low ability dimension (comprehensive application 66.7%), average process dimension.

Weak type (21 people): All indicators are below the mean, and the comprehensive application is only 51.6%. The process dimension is poor (homework completion rate 69.5%,

participation rate 2.24).

Table 3. Partial Student Membership Matrix (Partial)

Student	ID Excellent type	Potential type	Weak type
1	0.0091	0.9887	0.0022
2	0.8044	0.0471	0.1485
3	0.0468	0.9389	0.0143
4	0.0738	0.015	0.9113
5	0.9612	0.0246	0.0142
6	0.3037	0.6559	0.0404
7	0.0432	0.0141	0.9426
8	0.9875	0.0059	0.0066
9	0.0843	0.8873	0.0285
10	0.3401	0.0486	0.6113
...	...	...	...
85	0.9478	0.019	0.0331
86	0.8201	0.1402	0.0396

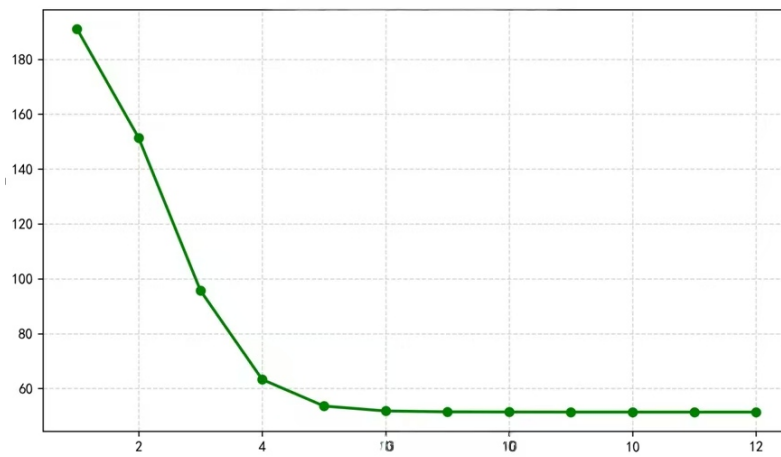


Fig. 1 The curve of the objective function decreasing with the number of iterations

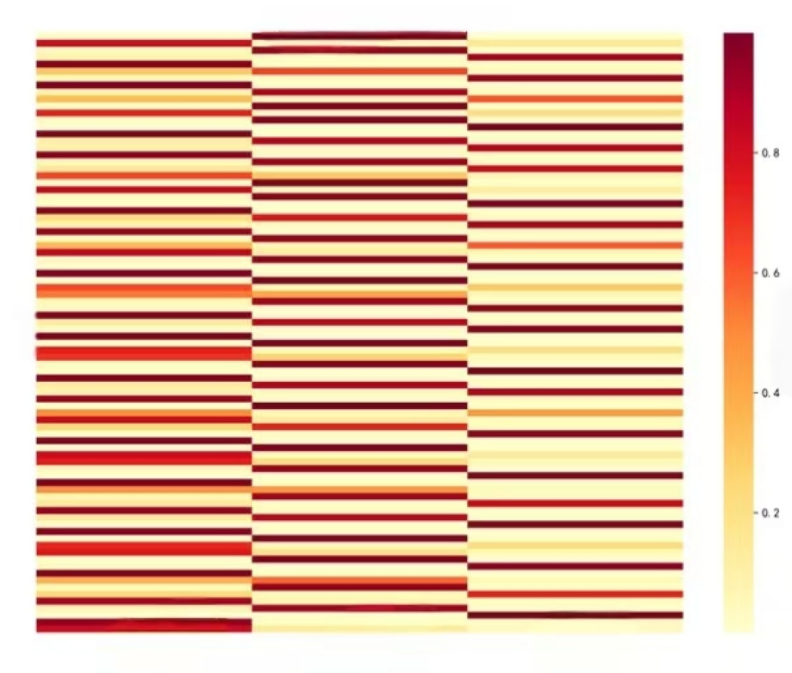


Fig. 2 Membership degree heatmap

Based on the membership degree heatmap, we can clearly see that dividing students into three types is reasonable.

3. Summary

Comparison with K-means hard clustering.

Run the K-means algorithm using the same data and $c=3$.

Compare the K-means results with the FCM results and calculate the Silhouette Coefficient.

Table 4. FCM and K-means contour coefficient comparison

Algorithm	Average silhouette coefficient	Class 1 contour coefficient	Class 2 contour coefficient	Class 3 contour coefficient
FCM	0.48	0.52	0.44	0.49
K-means	0.41	0.45	0.38	0.42

The average contour coefficient of FCM is 0.07 higher than that of K-means, indicating that its clustering results are more compact internally and have better inter class separation. In addition, FCM provides membership information that K-means cannot provide. For example, for borderline student ID 10, FCM provides (0.08, 0.62, 0.30), and teachers can determine that the student "mainly belongs to the potential type, but has a tendency to slide towards the weak type", and therefore take preventive interventions.

References

- [1] Qadir, A. M. (2023). Clustering algorithms in educational data mining: A review of applications and trends (2013–2023). *Education and Information Technologies*, 28, 14403-14445. <https://doi.org/10.1007/s10639-023-11756-6>
- [2] Hasan, M., Rahman, S., Hossain, M., & Khan, R. (2025). Clustering student performance data using K-means for adaptive learning strategies. *International Journal of Evaluation and Research in Education*, 14(1), 504-513.
- [3] Bustomi, Y., Lestari, R., Kusnadi, A., & Firdaus, A. (2022). Application of fuzzy C-means clustering for junior high school student grouping based on academic and behavioral indicators. *Journal of Physics: Conference Series*, 2394, Article 012016. <https://doi.org/10.1088/1742-6596/2394/1/012016>
- [4] Rustam, Z., & Usman, M. (2026). Fuzzy C-means clustering for interpretable learning trajectory modeling in outcomes-based education. *Barekeng: Jurnal Ilmu Matematika dan Terapan*, 20(1), 109-122.
- [5] Shi, Z. H. (2019). Research and practice on performance prediction based on multi-source data comprehensive analysis [Master's thesis]. Inner Mongolia University.
- [6] Xie, Y. T. (2019). Research on course correlation and student comprehensive quality evaluation based on academic performance [Master's thesis]. Central China Normal University.
- [7] Wang, S. X. (2018). Analysis of middle school students' performance based on fuzzy inference model [Master's thesis]. Minzu University of China.
- [8] Zhao, J. L. (2018). Comprehensive evaluation and application research on middle school academic performance based on fuzzy clustering analysis [Master's thesis]. Northwest Normal University.
- [9] Bezdek, J. C. (1981). Pattern recognition with fuzzy objective function algorithms. Plenum Press. <https://doi.org/10.1007/978-1-4757-0450-1>