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Generalized dissipativity state estimation of delayed neural networks under a novel event-triggered strategy

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Abstract: This paper investigates the generalized dissipativity state estimation problem for delayed neural networks (DNNs) under a novel event-triggered strategy. Unlike traditional Lyapunov-Krasovskii Functionals (LKFs) which impose positive definiteness constraints on matrices, this paper constructs a novel delay-dependent LKF. The positive definiteness constraint on the LKF matrix is relaxed via a quadratic inequality, which offers more flexibility in the functional construction. Meanwhile, to guarantee satisfactory estimator performance under limited communication resources, a novel adaptive event-triggered strategy (AETS) with memory-based triggering logic is proposed. Unlike existing event-triggered (ET) strategies, the proposed AETS can dynamically adjust triggering parameters and further reduce communication frequency. Accordingly, two sufficient criteria are derived via linear matrix inequalities (LMIs) to guarantee the dissipativity of the estimation error system. Finally, numerical examples are provided to demonstrate the superiority of the proposed methods.

Keywords: Delayed neural networks; Adaptive event-triggered; Generalized dissipativity state estimation; Stability.

1. Introduction

Neural networks (NNs), owing to their robust nonlinear modeling capabilities and ability to learn complex patterns, constitute an active and rapidly evolving research field, with applications in diverse domains such as control systems [3], computer vision [4], and robotics [5]. In practical scenarios, time delays arise due to the limited operating speed of signal processing devices and signal transmission among neurons. In some cases, time delays may cause the considered system to exhibit oscillations, instability, or even complete loss of control. For this reason, the measured output in large-scale NNs typically contains only partial neuron states, so numerous research works on state estimation have been published during the past few decades. Meanwhile, time delays are unavoidable in DNNs, so it is necessary to investigate the state estimation problem for NNs [6–8]. In addition, dissipativity refers to the property that the energy dissipated inside the system is less than the energy input from the outside. First introduced by reference [9], dissipativity theory essentially implies the existence of a non-negative storage function such that the energy supply rate of the system is always greater than the energy dissipation. For a given energy supply rate, a system is said to be dissipative if there exists a non-negative energy storage function dependent on the system states that satisfies the dissipative inequality. Based on this, investigating the dissipative state estimation of DNNs is a highly meaningful research topic.

Recently, due to the accelerating growth of telecommunication networks, many studies about state estimation of DNNs have attracted extensive attention. However, for the limited network resources in the output of DNNs to estimator, network traffic congestion is inevitable that can result in networked delays and packet loss [23]. Meanwhile, if state estimator is applied time triggered strategy in state estimation, all the measured output data are delivered to the estimator. The limited networked resources will be wasted to some extent. Regarding this phenomenon, event-triggered strategy (ETS) has attracted extensive attention, and abundant research results have been achieved. For example, reference [1] presents a dissipativity-based ETS consensus tracking control method for nonlinear leader-following multiagent systems with generally uncertain Markovian switching topologies and time-varying delays. Reference [2] introduced a finite-time extended dissipative fault estimation scheme for discrete-time Markov jump neural networks based on an ETS. And reference [2] presented an ETS H∞ state estimation method for continuous-time neural networks with time-varying delays, designing a new integral inequality and triggering scheme to balance estimation performance with network resource conservation. Consequently, how to design an ETS that can fully conserve resources and maintain desirable estimation performance deserves further study.

However, despite the inevitable conservatism of result, the LKF method is widely recognized as a crucial tool for time-delay system analysis. Therefore, designing appropriate LKFs with less conservatism is of paramount importance. Recent research has extensively focused on this challenge. To incorporate information of both time delay and system state as comprehensively as possible, many scholars proposed various types of LKF. For instance, delay-partitioning LKF [21], augmented LKF [20], switched LKF [22]. However, these advances have improved the acquisition of time-varying delay information in LKF, but the matrices involved usually need to satisfy strict positive definiteness constraints. To address this issue, in reference [14], a novel delay-dependent LKF is constructed that removes the positive-definiteness constraints on the augmented quadratic term and the delay-product-type terms. This construction simultaneously considers two double integral states into the single integral terms to reduce conservatism. Although the aforementioned methods can effectively reduce conservatism, there is still room to improve the degrees of freedom in LKF matrices.

Based on the above statement, this article focuses on the generalized dissipativity state estimation of DNNs. The contributions are as follows. 1) To further save resources and adjust the triggering parameters, a novel AETS is introduced.

And this strategy is based on the memory state neurons. 2) Then, an extended delay-dependent LKF, which incorporates quadratic terms about delay, is constructed to derive linear matrix inequalities. This LKF relaxes the requirement of positive definiteness constraints of matrices and enhances the degree of functional freedom compared to recent ones. Then two given examples illustrate the advantage of the presented strategy.

Notation: $\mathfrak{R}^n$ and $\mathfrak{R}^{n\times m}$ are the n -dimensional real vectors and $n\times m$ matrices. $\mathbb{S}_+^n$ represents the positive definite symmetric matrices. The symbol $*$ denotes the omission of symmetric components. $\hat{\vartheta}(t)$ stands for the estimated state of $\vartheta(t)$. $e_j = [0_{n\times n(j-1)} \quad I_n \quad 0_{n\times n(12-j)}]$, where $j=\{1,2,\dots,12\}$. Other notations are standard.

2. Preliminaries

Consider a DNN system with

$$\begin{cases}\dot{x}(t) = -Ax(t) + f\big(\kappa(t)\big) + B_1\varpi(t)\\ \kappa(t) = Wx\big(t-d(t)\big) + J\\ y(t) = Cx(t)\\ z(t) = L_0x(t)\\ x(t) = \Theta(t),\ t\in[-h,0)\end{cases} \tag{1}$$

where $x(t)\in\mathfrak{R}^n, \kappa(t)\in\mathfrak{R}^n, y(t)\in\mathfrak{R}^m, z(t)\in\mathfrak{R}^p$, and $\omega(t)\in L_2[0,\infty)$ represent the neuron state, the input of the activation function, the measurable output of the network, the vector to be estimated, and the disturbance, respectively; $A\in\mathfrak{R}^{n\times n}$ is a positive definite diagonal matrix; $W = [W_{ij}]_{n\times n}$ is a given delayed connection weight matrix; $J\in\mathfrak{R}^n$ is the exogenous input vector; C, B_1 and L_0 are matrices with compatible dimensions; $\Theta(t)$ is a given initial condition; $d(t), f(\cdot)\in\mathfrak{R}^n$ denotes the time-varying delay and the activation function, which satisfy

$$0\le d(t)\le h, \big|\dot{d}(t)\big|\le\mu<1,$$

$$0\le\frac{f_j(\iota_1)-f_j(\iota_2)}{\iota_1-\iota_2}\le l_j, \iota_1\ne\iota_2\in\mathfrak{R}, j=1,2,\cdots n \tag{2}$$

where h and μ are constants, and l_j are given constants and $f_j(0)=0$.

To reduce triggering frequency while preserving prescribed performance under limited communication resources, this paper proposes a novel AETS. Specifically, with a constant sampling period T , the measurable output $y(t)$ is sampled at instants $lT(l=0,1,\dots)$, yielding discrete samples $y(lT)$. The latest transmitted sample at a triggering instant is denoted as $y(t_kT)$. Let $\Phi=\{0,t_1T,t_2T,\dots,t_kT\}$ represent the set of transmission instants, where $\{t_1,t_2,\dots,t_k\}\subset\mathbb{N}$. For each interval $[t_kT,t_{k+1}T)$, define sub-interval endpoints $\Xi_{k,j}=(t_k+j)T$ with $j=0,1,\dots,j_m$ and $j_m=t_{k+1}-t_k-1$. The interval can then be expressed as $[t_kT,t_{k+1}T)=\bigcup_{j=0}^{j_m}\big[\Xi_{k,j},\Xi_{k,j+1}\big)$. Define $\tau(t)=t-\Xi_{k,j}$, and we have $0\le\tau(t)\le T$ and $\dot{\tau}(t)=1$. As stated [19], $\hat{y}\big(t-\tau(t)\big)$ is measurable. Then, the triggering instant is defined by the condition:

$$\begin{aligned}t_{k+1}T = t_kT + \min_{j\le1}\{jT|\ &[y(t_kT)-y(t_kT+jT)]^{\mathrm{T}}\Gamma[y(t_kT)\\ &- y(t_kT+jT)]\\ &>\sigma(t_kT)\big[y(t_kT)-y\big(t-\tau(t)\big)\big]^{\mathrm{T}}\Gamma\big[y(t_kT)\\ &- y\big(t-\tau(t)\big)\big]\big\},\end{aligned}$$

where $\vartheta_y(t)=y(t_kT)-y(lT), \Gamma>0$, and $\sigma(t_kT)$ is an adaptive parameter that satisfies

$$\begin{aligned}\sigma(t_{k+1}T) = \min\Big\{\underline{\sigma}, \max\Big\{\sigma(t_kT)\Big(1\\ -\frac{2}{\pi}\alpha\arctan\big(\beta\bar{e}(t_kT)\big)\Big),\overline{\sigma}\Big\}\Big\},\end{aligned}$$

with

$$\bar{e}(t_kT)=\sum_{i=0}^{n}\mu_i\,(\|y(t_{k-i+1}T)\|-\|y(t_{k-i}T)\|),$$

the weighting coefficients μ_i satisfy $\sum_{i=0}^{n}\mu_i=1$, and $0<\underline{\sigma}<\overline{\sigma}<1$. The scalars α and β are given positive constants.

Remark 1. Compared with the recent strategies in [2], [13],[17], this paper sets $\sigma(t_{k+1}T)$ to the interval $0<\underline{\sigma}<\overline{\sigma}<1$, where scalar parameters α and β are given. By considering that newly triggered data is generally more timely and important than previously triggered historical data, this paper further defines $\mu_i>\mu_{i+1}(i=1,2,\dots,n)$. In other words, the proposed AETS is designed based on the memory-based adaptive law that can utilize historical data to increase the transmission of effective data when the system's state response reaches the vertex. Meanwhile, the AETS can significantly reduce the transmission of unnecessary redundant data, ensuring the system maintains good performance while reducing triggering frequency.

A suitable state estimator is designed as:

$$\begin{cases}\dot{\hat{x}}(t) = -A\hat{x}(t) + f\big(\hat{\kappa}(t)\big) + K_1\Big(y(t_kT)-\hat{y}\big(t-\tau(t)\big)\Big)\\ \hat{\kappa}(t) = W\hat{x}\big(t-d(t)\big) + J + K_2\Big(y(t_kT)-\hat{y}\big(t-\tau(t)\big)\Big)\\ \hat{y}(t) = C\hat{x}(t)\\ \hat{z}(t) = L_0\hat{x}(t)\\ \hat{x}(t) = 0,\ t\in[-h,0)\end{cases} \tag{3}$$

Defining $\vartheta(t)=x(t)-\hat{x}(t)$ with (1) and (2), the error system can be constructed

$$\begin{cases}\dot{\vartheta}(t) = -A\vartheta(t) + f\big(\tilde{\kappa}(t)\big) - K_1C\vartheta\big(t-\tau(t)\big) - K_1\vartheta_y(t) + B_1\varpi(t)\\ \tilde{z}(t) = L_0\vartheta(t)\end{cases} \tag{4}$$

where $f\big(\tilde{\kappa}(t)\big)=f\big(\kappa(t)\big)-f\big(\hat{\kappa}(t)\big)$, and $\tilde{z}(t)=z(t)-\hat{z}(t)$.

Based on the S-procedure technique, for matrices $\Pi=\mathrm{diag}\{\pi_1,\pi_2,\dots,\pi_n\}$ satisfying $\pi_i>0(i=1,2,\dots,n)$, and a positive definite matrix $L=\mathrm{diag}\{l_1,l_2,\dots,l_n\}$, the following inequality holds

$$\begin{aligned}0\le & -2f^T\big(\tilde{\kappa}(t)\big)\Pi f\big(\tilde{\kappa}(t)\big)\\ & + 2f^T\big(\tilde{\kappa}(t)\big)\Pi L\big[-K_2C\vartheta\big(t-\tau(t)\big)\\ & + W\vartheta\big(t-d(t)\big) - K_2\vartheta_y(t)\big].\end{aligned}$$

Definition 1. For given real matrices $\Psi_0\ge0,\Psi_1\le0,\Psi_2$ and $\Psi_3\ge0$ satisfying that if $\Psi_0\ne0$ then $\Psi_1=0$ and $\Psi_2=0$, the filtering error system (4) is said to be generalized dissipative if under zero initial conditions, the following inequality holds for any $t_f\ge0$ and $w\in L_2[0,\infty)$:

$$\int_0^{t_f}Q\,(t)dt-\sup_{0\le t\le t_f}\tilde{z}^T(t)\Psi_0\tilde{z}(t)\ge0$$

where

$$Q(t)=\tilde{z}^T(t)\Psi_1\tilde{z}(t)+2\tilde{z}^T(t)\Psi_2\varpi(t)+\varpi^T(t)\Psi_3\varpi(t).$$

Lemma 1.[14] Long and Zhang and He and Wang and Wu (2022) Let $h>0$ and $f\big(d(t)\big)=\ a_2d(t)^2+a_1d(t)+a_0$, where $a_0,a_1,a_2\in\mathbb{R}$. It holds that $g\big(d(t)\big)<0$ for all $d(t)\in[0,h]$, if the following conditions hold:

$$g(0)<0,\ g(h)<0,\ g\left(\frac{i-1}{Z}h\right)-\frac{\alpha_{Zi}^2}{Z^2}h^2a_2 <0,\ g\left(\frac{i}{Z}h\right)-\frac{(1-\alpha_{Zi})^2}{Z^2}h^2a_2<0$$

where $\alpha_{Zi}\in[0,1], i=1,2,\dots,Z$, and Z is a positive integer.

Lemma 2.[18] Given an integer $k>0$, a scalar $\alpha\in(0,1)$, two matrices $\mathrm{Y}_1,\mathrm{Y}_2\in\mathbb{S}_+^n$, if there exist two symmetric matrices $\mathrm{M}_i,\ \mathrm{N}_i(i=1,2,\dots,k)$, and four matrices $\mathrm{X}_i(i=1,2)$ such that

$$\begin{bmatrix}\mathrm{Y}_1-\mathrm{M}(\alpha) & \mathrm{X}(\alpha)\\ * & \mathrm{Y}_2-\mathrm{N}(\alpha)\end{bmatrix}\geq 0,$$

then the subsequent inequality holds

$$\begin{bmatrix}\frac{1}{\alpha}Y_1 & 0\\ 0 & \frac{1}{1-\alpha}Y_2\end{bmatrix}\geq\begin{bmatrix}Y_1+\widehat{M}(\alpha) & X(\alpha)\\ * & Y_2+\widehat{N}(\alpha)\end{bmatrix}$$

where

$$\mathrm{M}(\alpha)=\sum_{i=1}^{k}\alpha\,(1-\alpha)^{i-1}\mathrm{M}_i,\widehat{\mathrm{M}}(\alpha)=\sum_{i=1}^{k}(1-\alpha)^i\,\mathrm{M}_i,$$

$$\mathrm{N}(\alpha)=\sum_{i=1}^{k}(1-\alpha)\,\alpha^{i-1}\ \mathrm{N}_i,\widehat{\mathrm{N}}(\alpha)=\sum_{i=1}^{k}\alpha^i\ \mathrm{N}_i$$

$$X(\alpha)=\alpha X_1+(1-\alpha)X_2$$

3. Main results

For convenience, we first define

$$\Omega(t)=\left[\dot{\vartheta}^T(t),\vartheta^T(t),\vartheta^T\big(t-d(t)\big),\vartheta^T(t-h),\vartheta^T\big(t-\tau(t)\big),\vartheta(t-T),\int_{t-d(t)}^{t}\vartheta^T\,(r)dr,\int_{t-h}^{t-d(t)}\vartheta^T\,(r)d,\vartheta_y^T(t),\varpi^T(t),f\big(\tilde{\kappa}(t)\big)\right]^T.$$

Theorem 1. For positive scalars $h,d,\mu,\varsigma_1,\varsigma_2,\varsigma_3$, and given matrix X , matrix $\Psi_0,\Psi_1,\Psi_2,\Psi_3$, and matrix $L=\mathrm{diag}\{l_1,l_2,\dots,l_n\}\in\mathbb{S}_+^n$, if there exist matrices $M_1,M_2,M_3\in\mathbb{R}^{n\times n}$, and R_1 , $R_2,W_1,W_2,Q_1,Q_2,P\in\mathbb{S}_+^n$, diagonal matrices $\Pi\in\mathbb{S}_+^n$, and matrices $G_i\in\mathbb{R}^{n\times m}(i=1,2)$ such that

$$\widetilde{\Delta}(0)<0,\ \widetilde{\Delta}(h)<0,\ \widetilde{\Delta}\left(\frac{i-1}{N}h\right)-\frac{\alpha_{Ni}^2}{N^2}h^2\hat{a}_0 <0,\ \widetilde{\Delta}\left(\frac{i}{N}h\right)-\frac{(1-\alpha_{Ni})^2}{N^2}h^2\hat{a}_0<0 \tag{5}$$

$$\begin{bmatrix}M_{3i} & \frac{1}{4}M_{2i}\\ \frac{1}{4}M_{2i} & \frac{1}{3}M_{1i}\end{bmatrix}+sym\left\{\begin{bmatrix}\mathbf{0} & E_{1i}\\ \mathbf{0} & E_{2i}\end{bmatrix}\right\}>0 \tag{6}$$

$$h^2\begin{bmatrix}\frac{1}{3}M_{1i} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}+h^2\mathrm{sym}\left\{\begin{bmatrix}-E_{1i} & \mathbf{0}\\ -E_{2i} & \mathbf{0}\end{bmatrix}\right\}+h\begin{bmatrix}\frac{1}{2}M_{2i} & \frac{1}{6}M_{1i}\\ \frac{1}{6}M_{1i} & \mathbf{0}\end{bmatrix}+h\mathrm{sym}\left\{\begin{bmatrix}-E_{1i} & E_{1i}\\ -E_{2i} & E_{2i}\end{bmatrix}\right\}+\begin{bmatrix}M_{3i} & \frac{1}{4}M_{2i}\\ \frac{1}{4}M_{2i} & \frac{1}{3}M_{1i}\end{bmatrix}+\mathrm{sym}\left\{\begin{bmatrix}\mathbf{0} & E_{1i}\\ \mathbf{0} & E_{2i}\end{bmatrix}\right\}>0 \tag{7}$$

$$-h^2\begin{bmatrix}\frac{1}{3}M_{1i} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}-h^2\mathrm{sym}\left\{\begin{bmatrix}-E_{1i} & \mathbf{0}\\ -E_{2i} & \mathbf{0}\end{bmatrix}\right\}+\begin{bmatrix}M_{3i} & \frac{1}{4}M_{2i}\\ \frac{1}{4}M_{2i} & \frac{1}{3}M_{1i}\end{bmatrix}+\mathrm{sym}\left\{\begin{bmatrix}\mathbf{0} & E_{1i}\\ \mathbf{0} & E_{2i}\end{bmatrix}\right\}>0 \tag{8}$$

where

$$\Omega^T(t)\Delta(t)\Omega(t)=\sum_{i=1}^{7}\Omega^T\,(t)\Delta_i(t)\Omega(t)$$

$$\Omega^T(t)\Delta_1(t)\Omega(t)=2e_1^T(d^2(t)M_1+d(t)M_2+M_3)e_2 +e_2^T(2d(t)M_1+M_2+W_1+W_2+R_1+h^2Q_1)e_2 -e_5^TPe_5-e_3^T\left(1-\dot{d}(t)\right)(R_1-R_2)e_3-e_4^TR_2e_4 -e_6^TW_2e_6+h^2e_1^TQ_2e_1$$

$$\Omega^T(t)\Delta_2(t)\Omega(t)=-e_8^T\left(Q_1+\sum_{i=1}^{2}\left(1-\frac{d(t)}{h}\right)^i\mathrm{M}_{1i}\right)e_8 -2e_8^T(\alpha X_{11}+(1-\alpha)X_{12})e_7 -e_7^T\left(Q_1+\sum_{i=1}^{2}\left(\frac{d(t)}{h}\right)^i\ \mathrm{N}_{1i}\right)e_7$$

$$\Omega^T(t)\Delta_3(t)\Omega(t)=-(e_3-e_4)^T\left(Q_2+\sum_{i=1}^{2}\left(1-\frac{d(t)}{h}\right)^i\mathrm{M}_{2i}\right)(e_3-e_4) -2(e_3-e_4)^T(\alpha X_{21}+(1-\alpha)X_{22})(e_2-e_3)\big) -(e_2-e_3)^T\left(Q_2+\sum_{i=1}^{2}\left(\frac{d(t)}{h}\right)^i\ \mathrm{N}_{2i}\right)(e_2-e_3)$$

$$\Omega^T(t)\Delta_4(t)\Omega(t)=2[e_2^T\varsigma_1+e_1^T\varsigma_2+e_9^T\varsigma_3X^T]F[-Ae_2+e_{11}-K_1Ce_5-K_1e_9+B_1e_{10}-e_1]$$

$$\Omega^T(t)\Delta_5(t)\Omega(t)=\sigma(Ce_5+e_9)^T\Gamma(Ce_5+e_9)-e_2^T\Gamma e_2$$

$$\Omega^T(t)\Delta_6(t)\Omega(t)=2e_{11}^T\Pi L(-K_2Ce_5+We_3-K_2e_9) -2e_{11}^T\Pi e_{11}$$

$$\Omega^T(t)\Delta_7(t)\Omega(t)=e_2^T\Psi_1e_2+2e_2\Psi_2e_{10}+e_{10}^T\Psi_3e_{10}$$

$e_j=[0_{n\times n(j-1)}\quad I_n\quad 0_{n\times n(12-j)}]$ and $j=\{1,2,\dots,11\}$,

and the corresponding gains are determined as $K_1=F^{-1}G_1,K_2=(\Pi L)^{-1}G_2$.

Proof. Select a suitable LKF for system (4)

$$V(t)=V_D(t)+\big(T-\tau(t)\big)\vartheta^T\big(t-\tau(t)\big)P\vartheta\big(t-\tau(t)\big) +\int_{t-d(t)}^{t}\vartheta^T\,(r)R_1\vartheta(r)dr+\int_{t-h}^{t-d(t)}\vartheta^T\,(r)R_2\vartheta(r)dr +\int_{t-\tau(t)}^{t}\vartheta^T\,(r)W_1\vartheta(r)dr+\int_{t-T}^{t}\vartheta^T\,(r)W_2\vartheta(r)dr +h\int_{t-h}^{t}\int_{s}^{t}\vartheta^T\,(r)Q_1\vartheta(r)drds +h\int_{t-h}^{t}\int_{s}^{t}\dot{\vartheta}^T\,(r)Q_2\dot{\vartheta}(r)drds$$

$$V_D(t)=\vartheta^T(t)(d^2(t)M_1+d(t)M_2+M_3)\vartheta(t)$$

Remark 2. In the construction of the delay-dependent LKF, the term $V_D(t)$ is designed to incorporate a quadratic dependence on the time-varying delay $d(t)$, expressed as $V_D(t)=\ \vartheta^T(t)(d^2(t)M_1+d(t)M_2+M_3)\vartheta(t)$. Unlike conventional LKFs that impose strict positive-definiteness constraints on constant matrices [2,10,11,12], this construction explicitly incorporates the delay $d(t)$ and $d^2(t)$ into the functional structure. The introduction of the quadratic term $d^2(t)M_1$ provides additional degrees of freedom, enabling a more precise characterization of $d(t)$. By utilizing the properties of $d(t)$ and applying Lemma 1, the positive definiteness of $V_D(t)$ can be ensured without requiring M_1, M_2, M_3 to be positive definite. This approach effectively relaxes the constraints on the LKF matrices and enhances the flexibility of LKF for system (4), thereby reducing conservatism in the derived criteria.

For a zero matrix0and identity matrix I with appropriate dimension, the auxiliary zero equation is obtained

$$\mathrm{sym}\left\{\left(d^2(t)\begin{bmatrix}-E_{1i} & \mathbf{0}\\ -E_{2i} & \mathbf{0}\end{bmatrix} + d(t)\begin{bmatrix}-E_{1i} & E_{1i}\\ -E_{2i} & E_{2i}\end{bmatrix} + \begin{bmatrix}\mathbf{0} & E_{1i}\\ \mathbf{0} & E_{2i}\end{bmatrix}\right)\eta(t)\right\} = \mathbf{0} \tag{9}$$

where $\eta^T(t) = [\mathbf{I} \quad d(t)\mathbf{I}]$ and free matrix $E_i = [E_{1i}, E_{2i}]^T$.

For (6), we define $l\big(d(t)\big)$ as $d^2(t)M_1 + d(t)M_2 + M_3$, and one has

$$l\big(d(t)\big) \triangleq \eta(t)\left(d^2(t)\begin{bmatrix}\frac{1}{3}M_{1i} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix} + d(t)\begin{bmatrix}\frac{1}{2}M_{2i} & \frac{1}{6}M_{1i}\\ \frac{1}{6}M_{1i} & \mathbf{0}\end{bmatrix} + \begin{bmatrix}M_{3i} & \frac{1}{4}M_{2i}\\ \frac{1}{4}M_{2i} & \frac{1}{3}M_{1i}\end{bmatrix}\right)\eta^T(t) \tag{10}$$

Combining (7) and (8), we have

$$l\big(d(t)\big) \triangleq \eta(t)\big(d^2(t)\widetilde{M}_{1i} + d(t)\widetilde{M}_{2i} + \widetilde{M}_{3i}\big)\eta^T(t) \tag{11}$$

where

$$\widetilde{M}_{1i} = \begin{bmatrix}\frac{1}{3}M_{1i} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix} + \mathrm{sym}\left\{\begin{bmatrix}-E_{1i} & \mathbf{0}\\ -E_{2i} & \mathbf{0}\end{bmatrix}\right\}$$

$$\widetilde{M}_{2i} = \begin{bmatrix}\frac{1}{2}M_{2i} & \frac{1}{6}M_{1i}\\ \frac{1}{6}M_{1i} & \mathbf{0}\end{bmatrix} + \mathrm{sym}\left\{\begin{bmatrix}-E_{1i} & E_{1i}\\ -E_{2i} & E_{2i}\end{bmatrix}\right\}$$

$$\widetilde{M}_{3i} = \begin{bmatrix}M_{3i} & \frac{1}{4}M_{2i}\\ \frac{1}{4}M_{2i} & \frac{1}{3}M_{1i}\end{bmatrix} + \mathrm{sym}\left\{\begin{bmatrix}\mathbf{0} & E_{1i}\\ \mathbf{0} & E_{2i}\end{bmatrix}\right\}$$

For $V_D(t)$ and Lemma 1 with $Z = 1$, we get $l\big(d(t)\big) > 0$ and hence $V_D(T) > 0$.

$$\begin{aligned}\dot{V}(t) &= 2\dot{\vartheta}^T(t)(d^2(t)M_1 + d(t)M_2 + M_3)\vartheta(t)\\ &+\vartheta^T(t)(2d(t)M_1 + M_2 + W_1 + W_2 + R_1 + h^2Q_1)\vartheta(t)\\ &- \vartheta^T\big(t-\tau(t)\big)P\vartheta\big(t-\tau(t)\big)\\ &- \big(1-\dot{d}(t)\big)\vartheta^T\big(t-d(t)\big)(R_1 - R_2)\vartheta\big(t-d(t)\big)\\ &- \vartheta^T(t-h)R_2\vartheta(t-h)\\ &-\vartheta^T(t-T)W_2\vartheta(t-T) + h^2\dot{\vartheta}^T(t)Q_2\dot{\vartheta}(t)\\ &- h\int_{t-h}^{t}\vartheta^T(r)Q_1\vartheta(r)dr\\ &-h\int_{t-h}^{t}\dot{\vartheta}^T(r)Q_2\dot{\vartheta}(r)dr\\ &= \Omega^T(t)\Delta_1\Omega(t) - h\int_{t-h}^{t}\vartheta^T(r)Q_1\vartheta(r)dr\\ &- h\int_{t-h}^{t}\dot{\vartheta}^T(r)Q_2\dot{\vartheta}(r)dr \end{aligned} \tag{12}$$

Utilizing Jensen inequality Lemma and the Lemma 2 with $k = 2$, it follows that:

$$\begin{aligned} &-h\int_{t-h}^{t}\vartheta^T(r)Q_1\vartheta(r)dr\\ &= -h\int_{t-h}^{t-d(t)}\vartheta^T(r)Q_1\vartheta(r)dr - h\int_{t-d(t)}^{t}\vartheta^T(r)Q_1\vartheta(r)dr\\ &\le -\frac{h}{d(t)}\left(\int_{t-h}^{t-d(t)}\vartheta(r)dr\right)^T Q_1\left(\int_{t-h}^{t-d(t)}\vartheta(r)dr\right)\\ &-\frac{h}{h-d(t)}\left(\int_{t-d(t)}^{t}\vartheta(r)dr\right)^T Q_1\left(\int_{t-d(t)}^{t}\vartheta(r)dr\right)\\ &\le -\left(\int_{t-h}^{t-d(t)}\vartheta(r)dr\right)^T\left(Q_1 + \sum_{i=1}^{2}(1-\alpha)^i\,\mathrm{M}_{1i}\right)\left(\int_{t-h}^{t-d(t)}\vartheta(r)dr\right)\\ &-2\left(\int_{t-h}^{t-d(t)}\vartheta(r)dr\right)^T(\alpha X_{11} + (1-\alpha)X_{12})\left(\int_{t-d(t)}^{t}\vartheta(r)dr\right)\\ &-\left(\int_{t-d(t)}^{t}\vartheta(r)dr\right)^T\left(Q_1 + \sum_{i=1}^{2}\alpha^i\,\mathrm{N}_{1i}\right)\left(\int_{t-d(t)}^{t}\vartheta(r)dr\right)\\ &= \Omega^T(t)\Delta_2(t)\Omega(t)\end{aligned} \tag{13}$$

Similar to (11), we have

$$\begin{aligned} &-h\int_{t-h}^{t}\dot{\vartheta}^T(r)Q_2\dot{\vartheta}(r)dr\\ &= -h\int_{t-h}^{t-d(t)}\dot{\vartheta}^T(r)Q_2\dot{\vartheta}(r)dr - h\int_{t-d(t)}^{t}\dot{\vartheta}^T(r)Q_2\dot{\vartheta}(r)dr\\ &\le -\left(\int_{t-h}^{t-d(t)}\dot{\vartheta}(r)dr\right)^T\left(Q_2 + \sum_{i=1}^{2}(1-\alpha)^i\,\mathrm{M}_{2i}\right)\left(\int_{t-h}^{t-d(t)}\dot{\vartheta}(r)dr\right)\\ &-2\left(\int_{t-h}^{t-d(t)}\dot{\vartheta}(r)dr\right)^T(\alpha X_{21} + (1-\alpha)X_{22})\left(\int_{t-d(t)}^{t}\dot{\vartheta}(r)dr\right)\\ &-\left(\int_{t-d(t)}^{t}\dot{\vartheta}(r)dr\right)^T\left(Q_2 + \sum_{i=1}^{2}\alpha^i\,\mathrm{N}_{2i}\right)\left(\int_{t-d(t)}^{t}\dot{\vartheta}(r)dr\right)\\ &= \Omega^T(t)\Delta_3(t)\Omega(t)\end{aligned} \tag{14}$$

where $\alpha = \frac{d(t)}{h}$.

Based on (4) and the AETS condition, one has

$$\begin{aligned}&2\big[\vartheta^T(t)\varsigma_1 + \dot{\vartheta}^T(t)\varsigma_2 + \vartheta_y^T(t)\varsigma_3X\big]F\big[-A\vartheta(t) + f\big(\tilde{\kappa}(t)\big)\\ &-K_1C\vartheta\big(t-\tau(t)\big) - K_1\vartheta_y(t) + B_1\omega(t) - \dot{\vartheta}(t)\big]\\ &= \Omega^T(t)\Delta_4(t)\Omega(t) = 0\end{aligned} \tag{15}$$

$$0 \le \sigma\Big(C\vartheta\big(t-\tau(t)\big) + \vartheta_y(t)\Big)^T\Gamma\Big(C\vartheta\big(t-\tau(t)\big) + \vartheta_y(t)\Big) - \vartheta^T(t)\Gamma\vartheta(t) = \Omega^T(t)\Delta_5(t)\Omega(t) \tag{16}$$

Next, the proof is completed from two aspects: 1) the estimation error system (4) is asymptotically stable with $\omega(t) = 0$ and 2) if zero initial conditions are satisfied, the estimation error system (4) is generalized dissipative.

Case I: When $\varpi(t) = 0$, noting $\Psi_1 \le 0$, from (19), one has

$$\dot{V}\big(x(t)\big) \leq \tilde{z}^{\mathrm{T}}(t)\Omega_1\tilde{z}(t) + \overline{\eta}^{\mathrm{T}}(t)\Theta\overline{\eta}(t) \leq \overline{\eta}^{\mathrm{T}}(t)\Theta\overline{\eta}(t) < 0 \qquad (17)$$

Clearly, when $\omega(t) = 0$, the error system (4) is asymptotically stable.

Case II: When $\varpi(t) = 0$, it follows from (19) that

$$\int_0^t Q\,(s)\mathrm{d}s \geq V(t) - V(0) \qquad (18)$$

If the zero-initial conditions are satisfied, $V(0) = 0$. Meanwhile, it comes from (19) and (14) that

$$V(t) \geq \vartheta^{\mathrm{T}}(t)P_1\vartheta(t) \geq \tilde{z}^{\mathrm{T}}(t)\Phi_0\tilde{z}(t) \qquad (20)$$

From (19) and (20), it is true that

$$\int_0^t Q\,(s)\mathrm{d}s \geq \tilde{z}^{\mathrm{T}}(t)\Psi_0\tilde{z}(t) \qquad (21)$$

(i) When $\omega_0 = 0$, for any $t_f \geq 0$, it follows from (21) that

$$\int_0^{t_f} Q\,(s)\mathrm{d}s \geq \tilde{z}^{\mathrm{T}}(t_f)\Psi_0\tilde{z}(t_f) = 0$$

which implies that (7) holds.

(ii) When $\omega_0 = 0$, from Definition 1, $\Psi_1 = 0, \Psi_2 = 0$, and $\Psi_3 > 0$, then $Q(t) = \omega^{\mathrm{T}}(t)\Psi_3\omega(t)$. For any $t_f \geq 0$, one can get

$$\int_0^{t_f} Q\,(s)\mathrm{d}s \geq \int_0^t Q\,(s)\mathrm{d}s \geq \tilde{z}^{\mathrm{T}}(t)\varpi_0\tilde{z}(t),\ 0 \leq t \leq t_f$$

Consequently,(7) also holds when $\omega_0 = 0$. It is clear from Definition 1 that the error system (4) is generalized dissipative.

Then the proof is completed.

From (12)-(16), the following equation can be expressed as a quadratic polynomial

$$\sum_{i=1}^{7} \Omega^T(t)\Delta_i(t)\Omega(t) = \tilde{a}_2 d^2(t) + \tilde{a}_1 d(t) + \tilde{a}_0$$
$$= \Omega^T(t)(\hat{a}_2 d^2(t) + \hat{a}_1 d(t) + \hat{a}_0)\Omega(t)$$
$$\overset{\text{def}}{=} \Omega^T(t)\tilde{\Delta}\big(d(t)\big)\Omega(t) \qquad (22)$$

where $\Omega^T(t)\Delta_6(t)\Omega(t)$ is defined in Theorem 1 and $\Omega^T(t)\Delta_7(t)\Omega(t) = Q(t)$.

By means of Lemma 1, if (5)-(8) are satisfied, it yields

$$\sum_{i=1}^{7} \Omega^T(t)\Delta_i(t)\Omega(t) < 0. \qquad (23)$$

From Definition 1, system (4) is under generalized dissipativity. Thus, the proof is completed

Remark 3. A generalized dissipativity criterion with AETS for DNNS is investigated in Theorem 1 by using delay-dependent LKF and Lemma 2. It is observed that the coefficient matrix $\hat{a}_2$ does not come from derivative of delay-dependent LKF but also from Lemma 2. Accordingly, the conservatism about DNNs is further reduced.

Next we introduce a Corollary without delay-dependent LKF to illustrate the effectiveness of the proposed method. Let $M_1 = M_2 = M_3 = M$.

Corollary 1. For positive scalars $h, d, \mu, \varsigma_1, \varsigma_2, \varsigma_3$, and given matrix X , matrix $\Psi_0, \Psi_1, \Psi_2, \Psi_3$, and matrix $L = \mathrm{diag}\{l_1, l_2, \dots, l_n\} \in \mathbb{S}_+^n$, if there exist matrices $M, R_1, R_2, W_1, W_2, Q_1, Q_2, P \in \mathbb{S}_+^n$, diagonal matrices $\Pi \in \mathbb{S}_+^n$, and matrices $G_i \in \mathbb{R}^{n\times m}(i = 1,2)$ such that

$$\widetilde{\Delta}(0) < 0,\ \widetilde{\Delta}(h) < 0,\ \widetilde{\Delta}\left(\frac{i-1}{N}h\right) - \frac{\alpha_{Ni}^2}{N^2}h^2\hat{a}_0 < 0,\ \widetilde{\Delta}\left(\frac{i}{N}h\right) - \frac{(1-\alpha_{Ni})^2}{N^2}h^2\hat{a}_0 < 0$$

where

$$\Omega^T(t)\Delta_1(t)\Omega(t) = 2e_1^T M e_2 + e_2^T(W_1 + W_2 + R_1 + h^2 Q_1)e_2 - e_5^T P e_5$$
$$-e_3^T\left(1 - \dot{d}(t)\right)(R_1 - R_2)e_3 - e_4^T R_2 e_4 - e_6^T W_2 e_6 + h^2 e_1^T Q_2 e_1$$

and other symbols are the same as Theorem 1.

4. Numerical example

Consider DNNs (1) with $A = \{1.06,1.42,0.88\}$ and

$$W = \begin{bmatrix} -0.32 & 0.85 & -1.36 \\ 1.10 & 0.41 & -0.5 \\ 0.42 & 0.82 & -0.95 \end{bmatrix}, L_0 = \begin{bmatrix} 1 & 0 & 0.5 \\ 1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & -0.5 & 0.6 \end{bmatrix}, B_1 = \begin{bmatrix} -5 & -1 & 1 \\ 0.8 & 2 & 0.1 \end{bmatrix}.$$

Select system parameters with $\zeta_1 = \zeta_2 = 1, \zeta_3 = 2, \underline{\sigma} = 0.2, \overline{\sigma} = 0.8, T = 0.01, h = 0.6$,

$$\mu = 0.2, L = 1.1I, \Psi_0 = \gamma_{\min}^2 I, \Psi_1 = \Psi_2 = 0, \Psi_3 = I, X = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \\ 0 & 0 \end{bmatrix}.$$

To demonstrate the superiority of introduced delay-dependent LKF, based on the above condition, $\gamma_{\min}$ can be calculated as 1.8410 and 3.6849 by Theorem 1 and Corollary 1, respectively. This numerical comparison clearly validates the advantage of the proposed LKF in yielding less conservative stability and dissipativity criteria for estimation of DNNs.

The effectiveness of AETS in this paper will be shown as follows. By using Theorem 1 and the MATLAB LMI solver toolbox, gains and corresponding matrices are obtained

$$K_1 = \begin{bmatrix} 0.3770 & 0.0058 \\ -0.0180 & 0.5324 \\ -0.0023 & 0.0067 \end{bmatrix},\ K_2 = \begin{bmatrix} -0.0496 & -0.0053 \\ 0.0146 & -0.2113 \\ -0.0168 & -0.0167 \end{bmatrix}, \text{etc.}$$

Under the initial conditions $\vartheta(0) = [-4,2,4], \kappa(t) = 0.4 + 0.2\sin(t), \omega(t) = e^{-t}$, $f(x) = 1.1\tanh(x)$, the simulation results are presented as follows: Figures 1,3 show the results under the AETS, while Figures 2, 4 display the corresponding results under the ETS in [2]. The estimation errors, event-triggered release instants, and release intervals are plotted in Figures 1-4, which indicate that the result of AETS reduces the triggering times by 25.6% compared to that of the ETS. The simulation results confirm the effectiveness and superiority of the proposed AETS.

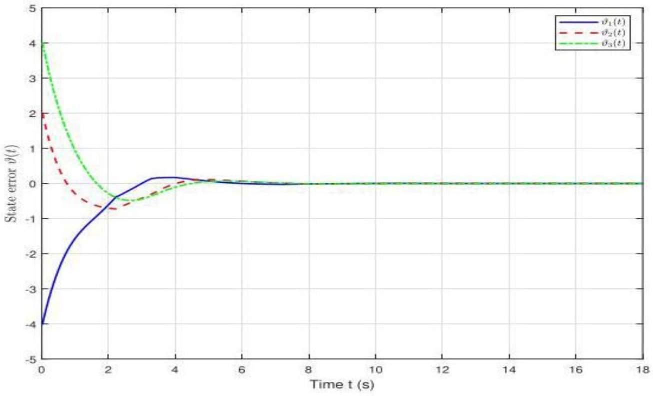

Figure 1. Variation curve of the estimation error $\vartheta(t)$ under AETS.

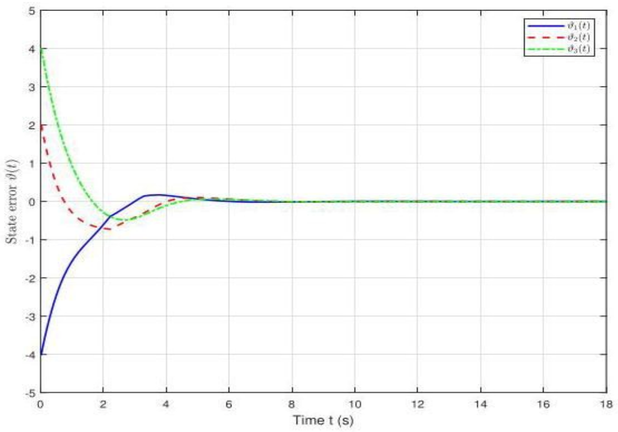

Figure 2. Variation curve of the estimation error $\vartheta(t)$ under ETS.

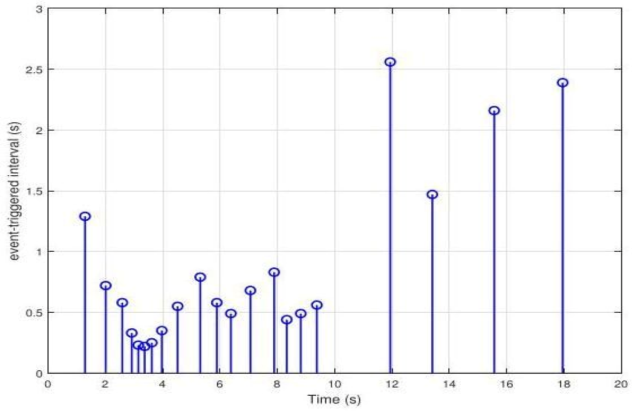

Figure 3. Triggering interval under AETS.

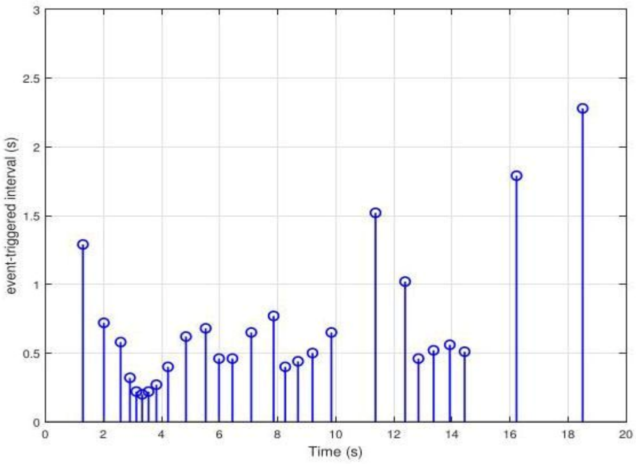

Figure 4. Triggering interval under ETS.

5. Conclusions

This paper has addressed the generalized dissipativity state estimation problem for DNNs under a novel event-triggered scheme. An AETS has been proposed to significantly reduce communication frequency while maintaining estimation performance. Furthermore, to relax the positive definiteness constraints on its matrices, a suitable delay-dependent LKF has been constructed via a quadratic inequality, thereby reducing conservatism. Based on this, sufficient linear matrix inequality conditions have been derived to guarantee the dissipativity of the estimation error system. As a result, the feasibility and advantage of the presented approach have been proved via some numerical simulations.

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