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Multi-Interference Missile Delivery Model Based on Particle Swarm Optimization Algorithm and Cross-Entropy Method

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Abstract: By deploying smoke-filled decoys to specific locations, enemy missiles can be disrupted, offering advantages such as low cost and high cost-effectiveness. This paper establishes effective shielding conditions by ensuring the distance from the point to the line segment is less than the spheres radius. It defines decision variables such as missile coordinates, drone speed, and direction, with the objective function being to maximize shielding duration. Constraints include time, speed, and direction angle. To solve the multi-objective model, the particle swarm optimization algorithm and cross-entropy method are applied based on different target requirements to determine the global optimal solution. Data validation confirms that the proposed model and solution method accurately calculate effective concealment duration, enabling the establishment of optimal smoke decoy deployment strategies.

Keywords: Horizontal Throw Motion; Model Optimization; PSO; CEM.

1. Introduction

With the rapid advancement of reconnaissance and precision strike technologies on modern battlefields, battlefield survivability has become a decisive factor in determining victory or defeat. Against this backdrop, smoke grenades—as an efficient, cost-effective passive jamming method—have seen their strategic value increasingly recognized. By rapidly generating extensive aerosol smoke screens, they effectively conceal friendly troop movements, disrupt enemy targeting systems (including optical and infrared-guided weapons), and buy crucial time for tactical deployment. However, the effectiveness of smoke screens is constrained by multiple factors such as wind direction, wind speed, deployment timing, and methods. Improper deployment strategies may inadvertently expose friendly intentions. Therefore, optimizing smoke grenade deployment strategies based on complex battlefield environments and tactical objectives to maximize interference effectiveness has become an urgent research topic. This paper aims to systematically analyze the key factors influencing smoke screen effectiveness and construct a scientific deployment strategy model to enhance the precision and reliability of battlefield smoke screen support [1][2].

2. Model assumptions

Assumption 1: Air resistance is neglected. The model assumes that after separation from the UAV, the smoke decoys motion is unaffected by air resistance. Consequently, the decoys trajectory is simplified to ideal projectile motion. Horizontally, it maintains the same velocity as when launched by the UAV, moving in uniform linear motion. Vertically, it undergoes free-fall motion under the sole influence of gravity.

Assumption 2: To simplify geometric calculations, the model treats all incoming missiles (M1, M2, M3) as point masses with no volume or size. This allows the missiles dimensions to be ignored in determining its spatial coordinates, enabling direct use of the coordinates provided in the problem.

Assumption 3: The detonation of the smoke grenade is assumed to occur instantaneously. Its descent velocity instantly changes from the terminal velocity of projectile motion to a constant 3 m/s, disregarding any time required for the explosion or velocity transition.

Assumption 4: Assume that upon receiving mission instructions, the drone can instantly adjust its flight direction without considering turning radius or deceleration/acceleration time. This simplifies the drones maneuvering process to an immediate direction change, followed by direct entry into uniform straight-line flight at constant altitude.

3. Model formulation and solution

3.1. Multi-Objective Cooperative Optimization Model Based on PSO Search

The missile moves at a constant velocity in a straight line toward the origin. $v_{bullet} = 300m/s = \left(\frac{3000}{\sqrt{101}}, \frac{300}{\sqrt{101}}\right)$. At time t, the coordinates of missile M1 are:

$$M\left(x_1, 0, z_1\right) = \left(20000 - \frac{3000}{\sqrt{101}}t, 0, 2000 - \frac{300}{\sqrt{101}t}\right) \quad (1)$$

Let FY1 have velocity v_1 and form angle α_1 with the negative x-axis, FY2 have velocity v_2 and form angle α_2 with the negative x-axis, and FY3 have velocity v_3 and form angle α_3 with the negative x-axis. FY1 releases a decoy at $t=t_1$ and detonates it at $t = t_2 + \Delta t_1$. FY2 releases a decoy at $t=t_2$ and detonates it at $t = t_2 + \Delta t_2$. FY3 releases a decoy at $t=t_3$ and detonates it at $t = t_3 + \Delta t_3$. Where $v_i \in [70,140], \alpha_i \in [0,2\pi), t_i \geq 0 (i=1,2,3)$.

For FY1, during $0 \leq t \leq t_1$, it undergoes uniform linear motion:

$$x = v_1 \cos\alpha_1 t_1$$
$$y = v_1 \sin\alpha_1 t_1 \quad (2)$$
$$z - 0$$

During $t_1 \le t \le t_1 + \Delta t_1$, it undergoes projectile motion [4]:

$$x = v_1 \cos\alpha_1 \Delta t_1$$
$$y = v_1 \sin\alpha_1 \Delta t_1 \quad (3)$$
$$z = \frac{1}{2} g \Delta t_1^2$$

During $t_1 + \Delta t_1 \le t \le t_1 + \Delta t_1 + 20$, it undergoes uniform deceleration:

$$z = 3\left(t - t_1 - \Delta t_1\right) \quad (4)$$

Therefore, the center coordinates of the FY1-deployed decoy cloud are:

$$C_1\left(x_2, y_2, z_2\right) = \left(17800 - v_1 \cos\alpha_1\left(t_1 + \Delta t_1\right), v_1 \sin\alpha_1\left(t_1 + \Delta t_1\right), 1800 - \frac{1}{2} g \Delta t_1^2 - 3\left(t - t_1 - \Delta t_1\right)\right) \quad (5)$$

Similarly, the center coordinates of the FY2-deployed chaff cloud are:

$$C_2\left(x_3, y_3, z_3\right) = \left(12000 - v_2 \cos\alpha_2\left(t_2 + \Delta t_2\right), 1400 - v_2 \sin\alpha_2\left(t_2 + \Delta t_2\right), 1400 - \frac{1}{2} g \Delta t_2^2 - 3\left(t - t_2 - \Delta t_2\right)\right) \quad (6)$$

Similarly, the center coordinates of the FY3-deployed chaff cloud are:

$$C_3\left(x_4, y_4, z_4\right) = \left(6000 - v_3 \cos\alpha_3\left(t_3 + \Delta t_3\right), -3000 + v_3 \sin\alpha_3\left(t_3 + \Delta t_3\right), 700 - \frac{1}{2} g \Delta t_3^2 - 3\left(t - t_3 - \Delta t_3\right)\right) \quad (7)$$

Similarly, according to the requirements, each of the three drones must deploy chaff that satisfies the effective shielding conditions. This allows us to calculate the effective shielding time interval T_i for each cloud cluster C_i.

The objective function is set as:

$$\max\left(d_1 + d_2 + d_3 - \left|d_1 \cap d_2\right| - \left|d_2 \cap d_3\right| - \left|d_1 \cap d_3\right| - \left|d_1 \cap d_2 \cap d_3\right|\right) \quad (8)$$

Here, $d_i = \left|T_i\right|$ represents the effective shielding duration of the i-th missile.

Speed Constraint:

$$v_i \in \left[70, 140\right], \left(i = 1, 2, 3\right) \quad (9)$$

Azimuth Constraint:

$$\alpha_i \in \left[0, 2\pi\right), \left(i = 1, 2, 3\right) \quad (10)$$

Time constraints:

$$t_i \ge 0, \Delta t_i \ge 0 \left(i = 1, 2, 3\right) \quad (11)$$

In summary, the optimization model established in this paper is:

$$\begin{cases} \max\left(d_1 + d_2 + d_3 - \left|d_1 \cap d_2\right| - \left|d_2 \cap d_3\right| - \left|d_1 \cap d_3\right| - \left|d_1 \cap d_2 \cap d_3\right|\right) \\ v_i \in \left[70, 140\right], \left(i = 1, 2, 3\right) \\ \alpha_i \in \left[0, 2\pi\right), \left(i = 1, 2, 3\right) \\ t_i \ge 0, \Delta t_i \ge 0 \left(i = 1, 2, 3\right) \\ M\left(x_1, 0, z_1\right) = \left(20000 - \frac{3000}{\sqrt{101}} t, 0, 2000 - \frac{3000}{\sqrt{101}} t\right) \\ C_1\left(x_2, y_2, z_2\right) = \left(17800 - v_1 \cos\alpha_1\left(t_1 + \Delta t_1\right), v_1 \sin\alpha_1\left(t_1 + \Delta t_1\right), 1800 - \frac{1}{2} g \Delta t_1^2 - 3\left(t - t_1 - \Delta t_1\right)\right) \\ C_2\left(x_3, y_3, z_3\right) = \left(12000 - v_2 \cos\alpha_2\left(t_2 + \Delta t_2\right), 1400 - v_2 \sin\alpha_2\left(t_2 + \Delta t_2\right), 1400 - \frac{1}{2} g \Delta t_2^2 - 3\left(t - t_2 - \Delta t_2\right)\right) \\ C_3\left(x_4, y_4, z_4\right) = \left(6000 - v_3 \cos\alpha_3\left(t_3 + \Delta t_3\right), -3000 + v_3 \sin\alpha_3\left(t_3 + \Delta t_3\right), 700 - \frac{1}{2} g \Delta t_3^2 - 3\left(t - t_3 - \Delta t_3\right)\right) \end{cases} \quad (12)$$

To solve the optimization problem of coordinated jamming against incoming missile M1 by deploying one smoke grenade from each of three drones (FY1, FY2, FY3), we aim to design an optimal strategy encompassing each drones flight direction, velocity, and the timing of smoke grenade deployment and detonation. This strategy seeks to maximize the total effective duration of shielding for the real target. This constitutes a complex high-dimensional cooperative optimization problem with 12 decision variables (4 per UAV: flight direction angle ψ, flight speed v_{uav}, release preparation time $t_{release}$, fuse time t_{fuse}). Given the problems high nonlinearity and intricate constraints, we employ a particle swarm optimization (PSO) algorithm for solution [4].

Step 1: Constructing a Multi-Objective Collaborative Optimization Model

Unlike the previous questions, the core of the fourth question lies in "collaboration." This means not only pursuing the coverage effect of a single smoke grenade but also optimizing the temporal coordination among three smoke grenades to maximize the total coverage duration. To achieve this, we designed a comprehensive objective function to guide the particle swarm optimization algorithm in finding the optimal solution. This function not only prioritizes the union duration of effective concealment provided by all three smoke grenades as the primary optimization goal but also incorporates multiple reward and penalty terms to ensure the strategys comprehensive effectiveness and rationality. The constructed optimization objective function O_{bj} is as follows:

$$O_{bj} = W_{union} \cdot T_{union} + \sum Rewards - \sum Penalties \quad (13)$$

Among these, T_{union} represents the total duration of the union of effective coverage intervals for the three smoke grenades on the absolute timeline.

$\sum Rewards$ include: A reward for the total effective duration of all deployed drones, incentivizing the utilization of each unit; - A reward for the shortest effective duration of any single drone, preventing strategic weaknesses; - A reward for the number of drones successfully achieving effective coverage, ensuring coordination.

Penalties $\sum Penalties$ are applied to deter undesirable strategies, specifically including: 1. Time Overlap Penalty: Imposes penalties on overlapping periods of effective smoke screen coverage to encourage complementary timing rather than redundant coverage. 2. Geometric Position Penalty: At the moment of detonation, if the center of the smoke cloud formed by the smoke grenade is not located on the line connecting the missile and the protected target, or deviates too far from this "interference corridor," a severe penalty is imposed to ensure effective spatial concealment. 3. Timing Rationality Penalty: Strategies where detonation occurs later than the missiles impact time are penalized to guarantee the practical significance of the interference. Additionally, to guarantee the effectiveness of a single interference event, strategies where the shielding duration of a single smoke grenade falls below a preset minimum threshold will also be penalized. 4. Single-Grenade Effect Penalty: If the effective shielding duration of a single smoke grenade becomes discontinuous (i.e., split into multiple segments), a minor penalty will be applied to encourage more stable and continuous single-event shielding effects.

Step 2: Global Optimization Using an Enhanced Particle Swarm Optimization (PSO) Algorithm

This paper employs a particle swarm optimization (PSO) algorithm to perform global optimization within a solution space comprising 12 decision variables. To enhance the algorithms efficiency and effectiveness, we implemented targeted improvements: During the initialization phase, in addition to randomly generating partial initial solutions, we adopted a "directed seeding" strategy. This strategy estimates the reference heading toward the target area and a reasonable time window ensuring the smoke grenade detonates ahead of the missile based on the initial relative positions of the UAV and target. Using this information, we generate a subset of high-quality initial particles, significantly accelerating the algorithms convergence speed. The iterative optimization process follows the mechanism described in Question 2, guiding the entire particle swarm toward the optimal solution region through continuous evaluation function calls.

Step 3: Precise Evaluation of Joint Obfuscation Effectiveness

When assessing the performance of each particle (i.e., each set of cooperative strategies), the core component is calculating the effective duration of a single smoke grenade. This calculation process aligns with the "rigorous caliber" high-precision numerical simulation method detailed in the first questions solution steps, whi ch is based on target discretization and adaptive time scanning. Building upon this foundation, the key challenge here lies in quantifying the synergistic effect of the three smoke grenades. We first convert the independent effective concealment time intervals of each smoke grenade onto a unified absolute mission timeline based on their respective detonation times. Subsequently, by performing set operations on this set of absolute time intervals, we obtain the total union duration T_{union} and the overlapping durations between them. By integrating the geometric and temporal parameters of the strategy, we can calculate the composite objective function score defined in the first step, thereby providing the basis for the next iteration of the particle swarm.

Step 4: Determination and Output of Optimal Strategy

After thorough iterative computation, the particle swarm optimization algorithm ultimately converges to a globally optimal solution. The corresponding 12 decision variables collectively form the optimal coordinated deployment strategy for Problem 4. We analyze this optimal solution to extract the specific motion parameters for each UAV, the deployment and detonation coordinates for smoke grenades, and their respective effective interference durations.

The effective joint shielding time for the three aircraft obtained from the above solution process is: 8.609122 s. The specific deployment strategy is shown in Table 1 below.

Table 1. Deployment Strategy for Three Drones and One Jammer

Drone			Drop-off point			Flashpoint			Effective interference duration
No.	Direction	Speed	x	y	z	x	y	z	
FY1	180	139.222	17353.001	0	1800	16614.251	0	1662.032	2.843529
FY2	198.739	137.899	9418.639	524.32	1400	7984.745	37.898	809.234	2.948417
FY3	105.598	109.328	5311.106	-532.319	700	5133.168	105.073	520.468	2.817176

Note: The x-axis is positive in the counterclockwise direction, with values ranging from 0 to 360 (degrees).

3.2. Collaborative Interference Optimization Model Based on CEM Global Optimization

For a missile moving with a constant velocity of $v_0 = 300m/s$ toward the origin, the position of the missile M_k changes with time as follows:

$$M_k(t) = M_k^0 - \frac{M_k^0}{\left\|M_k^0\right\|} v_0 t, (k = 1,2,3) \tag{14}$$

Where M_k^0 denotes the initial position of missile M_k.

Let drone FYi move with constant velocity v_i at an angle α_i to the negative x-axis (corresponding direction vector d_i, $\|d_i\| = 1$). Then the position of drone FYi varies with time as follows:

$$FYi(t) = FYi_0 + d_i v_i t \tag{15}$$

Among them, $v_i \in [70,140], \alpha_i \in [0,2\pi), i = 1,2,\cdots,5$

Let the time at which the i-th drone releases the j-th decoy be $t = t_{ij}, i = 1,2,\cdots,5$. Then the release position is $FYi(t_{ij})$.

For the j-th decoy of the i-th drone, if the detonation time is $t = t_{ij} + \Delta t_{ij}, i = 1,2,\cdots,5$, then the detonation position is $FYi\left(t_{ij} + \Delta t_{ij}\right)$.

During the time interval $t_{ij} + \Delta t_{ij} \le t \le t_{ij} + \Delta t_{ij} + 20$, as the cloud mass descends, the center position of the cloud mass for the j-th decoy launched by the i-th drone changes over time as follows:

$$C_{ij}\left(t\right) = FYi\left(t_{ij} + \Delta t_{ij}\right) - \left(0,0,3\right) \cdot \left(t - t_{ij}\Delta t_{ij}\right) \quad (16)$$

Require 5 drones, each capable of deploying up to 3 decoy flares, to jam 3 missiles. The maximum effective jamming duration is defined as the period during which at least one missiles line of sight to the actual target is obscured by cloud cover or smoke screen. Determine the duration that satisfies the effective jamming condition and maximize this duration.

From the preceding text, we can derive the effective shielding duration T_{ij}^k of the cloud cluster formed by the j-th chaff from the i-th drone against missile M_k. The objective function is:

$$\max\left(\sum T_{ij}^k\right) \quad (17)$$

Speed Constraint:

$$v_i \in \left[70,140\right] \quad (18)$$

Angle Constraint:

$$\alpha_i \in \left[0,2\pi\right) \quad (19)$$

Each drone may deploy a maximum of 3 decoy flares:

$$0 \le j \le 3 \quad (20)$$

Time Constraint:

$$t_{ij} \ge 0 \quad (21)$$

Each drone must deploy two decoy flares with a minimum interval of 1 second:

$$t_{i,j+1} \ge 1 \quad (22)$$

Detonation occurs after deployment:

$$\Delta t_{ij} \ge 0 \quad (23)$$

To address the aforementioned multi-to-multi cooperative interference optimization problem, we designed and implemented a computational solution framework based on global optimization algorithms. This problem features numerous decision variables, and the objective function calculation involves complex three-dimensional spatial geometric judgments, exhibiting highly nonlinear and multimodal characteristics. To effectively solve this problem, we adopted a step-by-step strategy, with the specific steps as follows:

Step 1: Problem Transformation and Objective Function Construction

First, we transform the problem into a high-dimensional parameter optimization problem. The decision variables comprise all drone flight parameters and all decoy deployment and detonation control parameters. The objective function is defined as the total union length of time intervals during which any missiles line of sight is effectively obscured within the specified total duration.

To render this objective function computable, we adopted the previously constructed occlusion detection model. Specifically, we discretized the surface of the true target cylinder into a dense cloud of sampled points. At each discrete time step, we computed the positions of all three missiles alongside the centers of all smoke clouds that had detonated and remained within their effective duration. A missile is deemed “fully effectively obscured” at a given time step only if the line-of-sight segment from any of its points toward all surface sampling points of the target intersects with at least one smoke cloud. By iterating through all time steps and accumulating the step lengths satisfying this condition, we compute the total effective obscuration duration corresponding to any given set of decision variables.

To facilitate efficient search by optimization algorithms, we adopted a decision variable parameterization and normalization method similar to previous approaches. All 40 decision variables were uniformly encoded into a one-dimensional vector, with each elements value range confined between [0, 1]. During evaluation, these normalized values are decoded and remapped back to their actual physical ranges. For complex temporal constraints, such as “each drone must deploy two decoys with a minimum interval of 1 second,” we employ a programmatic approach involving sorting and correction logic during decoding. This ensures that all generated strategies inherently satisfy the specified constraints.

Step 3: Global Optimization Using the Cross-Entropy Method (CEM)

Given the problems high complexity and non-convexity, traditional optimization methods like gradient descent prove impractical. We employ an efficient global optimization algorithm—the Cross-Entropy Method (CEM) [5] to search for the optimal combination of decision variables. CEM is an iterative optimization algorithm based on probability distributions, with the following core principles:

Initialization: First, we set initial mean and standard deviation for the probability distributions (Gaussian distributions) of the 40 decision variables. To accelerate convergence, initial means are set based on prior knowledge—for example, setting the drones initial heading to roughly face the target area.

Sampling and Evaluation: In each iteration, we randomly sample a batch of candidate solution vectors from the current multidimensional Gaussian distribution. For each candidate solution, we convert it into a specific deployment strategy through the decoding process in Step 2 and calculate its corresponding total effective coverage duration using the evaluation function constructed in Step 1.

Elite Selection and Update: Based on evaluation scores, we select the best-performing solutions from the current batch as “elite samples.” We then compute the mean and standard deviation of these elite samples, using these values to update the Gaussian distribution parameters for the next iteration.

By repeatedly executing the “sample-evaluate-update” cycle, the probability distribution of the entire solution gradually converges toward the region containing the global optimum. To prevent getting stuck in local optima, we also employ a multiple restart strategy: running the CEM algorithm independently multiple times and selecting the optimal result from these independent runs.

Step 4: Tiered Evaluation Acceleration and Final Solution Refinement

During the massive evaluation process of CEM algorithm iterations, we adopted a two-tier evaluation strategy-“coarse search followed by refinement”-to balance computational efficiency and accuracy. During the optimization iteration phase, rapid evaluations are conducted using coarsely discretized parameters. Once the algorithm converges to the optimal decision vector, a high-precision parameter set is applied to perform a detailed verification calculation on this solution, yielding the accurate total effective jamming duration. Finally, through post-processing analysis of the optimal strategy, we independently calculated the jamming contribution of each chum-chum to each missile. This enabled us to assign the optimal jamming target to each chum-chum and generate the final detailed deployment strategy table.

Based on the above solution steps, the specific deployment strategy is obtained, as shown in Table 2 below.

Table 2. Deployment Strategy for Five Drones and Many Jammer (Part)

Drone			Drop-off point				Flashpoint			Effective interference duration	Jammer No. of Missile
No.	Direction	Speed	No.	x	y	z	x	y	z		
FY1	156.297	70	1	17800	0	1800	17777.65	9.812	1799.404	3.25	M1
FY1	156.297	70	2	17280.98	227.861	1800	17252.73	240.263	1799.047	0	M1
FY1	156.297	70	3	16723.89	472.439	1800	16594.25	529.35	1779.937	0	M1
FY2	249.77	118.251	1	12000	1400	1400	11740.97	697.129	1203.173	0	M1
FY2	249.77	118.251	2	11778.72	799.557	1400	11624.27	380.445	1330.017	2.9	M2
FY2	249.77	118.251	3	11737.83	688.6	1400	11654.98	463.781	1379.863	0	M1
FY3	175.248	106.87	1	6000	-3000	700	5091.704	-2924.49	343.244	0	M1
FY3	175.248	106.87	2	5108.404	-2925.88	700	4899.625	-2908.53	681.151	0	M1
FY3	175.248	106.87	3	4380.405	-2865.36	700	4056.349	-2838.42	654.589	0	M1
FY4	275.904	112.885	1	11050.36	1513.013	1800	11082.56	1201.645	1762.283	0	M1
FY4	275.904	112.885	2	11077.87	1246.992	1800	11142.44	622.668	1648.362	0	M1
FY4	275.904	112.885	3	11092.27	1107.797	1800	11114.72	890.723	1781.668	0	M1
FY5	207.286	111.692	1	12434.33	-2291.79	1300	12018	-2506.55	1213.714	0	M1
FY5	207.286	111.692	2	11882.96	-2576.21	1300	11646.34	-2698.27	1272.127	0	M1
FY5	207.286	111.692	3	10232.78	-3427.44	1300	9906.72	-3595.63	1247.077	0	M1

4. Conclusion

The strength of this model lies in its rigorous mathematical construction combined with advanced computational strategies. Rooted in solid mathematical theory, it precisely defines the core “effective masking condition” through solid geometry—specifically, determining whether the line segment connecting the missile and target intersects the smoke cloud sphere. This clear geometric and algebraic formulation establishes a comprehensive theoretical foundation for subsequent complex optimization problems. For problems of varying complexity, the paper innovatively employs matching intelligent optimization algorithms. For single-objective optimization, the classical Particle Swarm Optimization (PSO) algorithm is used for global optimization. When the problem escalates to complex scenarios involving multiple variables and constraints, the model further introduces more powerful global optimization tools such as Differential Evolution (DE) and Cross-Entropy Method (CEM). This capability to select the most suitable algorithm based on problem characteristics not only effectively addresses the challenges posed by the rapid increase in decision variable dimensions but also significantly enhances optimization efficiency and the likelihood of finding the global optimum solution. However, this paper simplifies numerous factors such as missile volume to address computational complexity, omitting the influence of air resistance and related elements. Consequently, the calculated results deviate from actual conditions.

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